1/20/15 week 5 stat209 Math notes, part 2

# Regression Adjustment in Quasi-experiments

STAT 209

outcome $Y$
group $G = 1, 0$

Head Start
Boyle, Weisberg
in Oakes

mis-specified model $Y = \beta_0 + \beta_1 G + u$ t-test.
but $G, u$ not indep

adjustment (pretest score) $X$ (pre-test?)

$\hat{\alpha} = \bar{Y}_1 - \bar{Y}_0 - \hat{\beta}(\bar{X}_1 - \bar{X}_0)$

estimated effect

IV alternative
X as instrument for G to correct bias in $\hat{\beta}_{YG}$ (t-test)

$\hat{\beta} = 0$ t-test no adj.

$\hat{\beta} = 1$ gain scores if X "pretest"

$\hat{\beta} = \hat{\beta}_{YX \cdot G}$ standard ancova
(underadjusts, overadjusts)

correcting for underadjusts

$\hat{\beta} = \hat{\beta}_{YX \cdot G} / rel(X)$ adjustment for measurement error in slope

$\hat{\beta} = \hat{\beta}_{Y|X \cdot G} / r_{YX \cdot G} = \frac{S_{Y \cdot G}}{S_{X \cdot G}}$ validity correction
Campbell-Erlebacher
St. change score
(week 9)

$\hat{\beta} = \hat{\beta}_0$ control slope
Belson equiv to $D(\bar{X}_1)$
adjustment

Belson, W. A. (1956), "A technique for studying the effects of a television broadcast," Applied Statistics, 5, 195–202.

numerical illustrations

Anderson et al (1980) Ch. 12 Table 12.1 Head Start Data

| | Pre | Post | $r_{prepost}$ | n |
|---|---|---|---|---|
| Innovative curriculum | 17.1 (6.1) | 23.3 (4.6) | .67 | 157 |
| Standard Head start | 14.6 (6.2) | 18.9 (5.8) | .78 | 669 |

pre diff 2.5 post diff 4.4

Inference?

$\hat{\beta}$ options

| | | |
|---|---|---|
| t-test | $\hat{\beta}=0$ | $\hat{\alpha}=4.4$ |
| gain | $\hat{\beta}=1$ | $\hat{\alpha}=1.9$ |
| ancova | $\hat{\beta}=\beta_{Y1\cdot G}=.76$ | $\hat{\alpha}=2.5$ |
| C-E | $\hat{\beta}=\frac{5.6}{6.2}=.9$ | $\hat{\alpha}=2.1$ |
| Belson | $\hat{\beta}=.73$ | $\hat{\alpha}=2.57$ |

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## Analytic results

Weisberg (1979) Ancova bias positive or negative

Potential outcomes setup: $W_i$ outcome if T ($Q=1$); $Z_i$ outcome if C ($Q=0$)

$\alpha_i = W_i - Z_i$ treatment effect.

observable $Y_i = Z_i + Q_i\alpha$ (set $\alpha_i = \alpha$) $P = E(Q)$ prop in T

for non-random assignment ($\rho_{ZQ} \neq 0$)

$$\mu_{Y_1} - \mu_{Y_0} = \alpha + (\mu_{Z_1} - \mu_{Z_0})$$ ← selection bias

p 115 result. Can ancova with covariate X reduce or eliminate bias?

bias from ancova $\delta = (\mu_{Z_1} - \mu_{Z_0})\left(\frac{\rho_{ZQ\cdot X}}{\rho_{ZQ}}\right)\left(\frac{\sqrt{1-\rho_{ZX}^2}}{\sqrt{1-\rho_{XQ}^2}}\right)$

H.I.W uses $\pi = \delta/(\mu_{Z_1} - \mu_{Z_0})$ "proportion bias"

Table 1 charts possible patterns of bias.

# Weisberg Ancova Results

STAT 209
Week 5

Potential Outcomes
person $i$

$W_i$ outcome if T ($Q=1$)
$Z_i$ outcome if C ($Q=0$)

Recast week 2
Holland
treatment/control
difference
$\alpha_i = W_i - Z_i$
(set $\alpha_i = \alpha$)

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observable $Y_i = Z_i + Q_i \alpha$
$P = E(Q)$ prop in T

Non-random assignment $\rho_{ZQ} \neq 0$

observables
point-biserial $\rho_{ZQ}$

$\mu_{Y_1} - \mu_{Y_0}$

$\mu_{Z_1} - \mu_{Z_0} = \frac{\sigma_Z}{\sqrt{P(1-P)}} \rho_{ZQ}$

$= \alpha + (\mu_{Z_1} - \mu_{Z_0})$ selection bias

recall week 2 FACE results BIAS $= E(Y_c|S=t) - E(Y_c|S=$

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Ancova with covariate $X$, saves the day?
[note: matching works, eqs 9-12]

(18) $E(Y|X,Q) = \mu + \beta X + (\delta + \alpha) Q$
observables

can only estimate $\delta + \alpha$, want $\alpha$

bias from ancova

(19) $\delta = \rho_{ZQ \cdot X} \frac{\sigma_{Z \cdot X}}{\sigma_{Q \cdot X}}$

$= \rho_{ZQ \cdot X} \frac{\sigma_Z}{\sqrt{P(1-P)}} \frac{\sqrt{1-\rho_{XZ}^2}}{\sqrt{1-\rho_{XQ}^2}}$

proportion of bias

$\Pi = \frac{\delta}{\mu_{Z_1} - \mu_{Z_0}} = \frac{\rho_{ZQ \cdot X}}{\rho_{ZQ}} \frac{\sqrt{1-\rho_{XZ}^2}}{\sqrt{1-\rho_{XQ}^2}}$

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ancova can overadjust or increase bias

WEISBERG

Table 1
*Range of π for Different Combinations of $\rho_{ZQ}$, $\rho_{XZ}$, $\rho_{ZX}$*

| Basic situation | Case | Sign ($\rho_{ZQ}$) | Sign ($\rho_{XQ}$) | Sign ($\rho_{ZX}$) | π |
|---|---|---|---|---|---|
| 1 | 1 | + | + | + | $-\infty$ to $+1$ |
| | 2 | + | − | − | $-\infty$ to $+1$ |
| | 3 | − | + | − | $-\infty$ to $+1$ |
| | 4 | − | − | + | $-\infty$ to $+1$ |
| 2 | 5 | − | − | − | 1 to $+\infty$ |
| | 6 | − | + | + | 1 to $+\infty$ |
| | 7 | + | − | + | 1 to $+\infty$ |
| | 8 | + | + | − | 1 to $+\infty$ |

Case 1 right direction, but can overadjust

Case 2 adjustment in wrong direction