

Multiple Regressions for Multi-level Contextual Estimates

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a) regress Y on $\bar{X}, X - \bar{X}$ over all cases

$\bar{X}, X - \bar{X}$ orthogonal
"Cronbach" regression

obtain

$$\beta_{Y\bar{X} \cdot (X-\bar{X})} = \beta_{Y\bar{X}}^b$$

between

$$\beta_{Y(X-\bar{X}) \cdot \bar{X}} = \beta_{Y(X-\bar{X})} = \beta_{YX}^{w-p}$$

within pooled $Y - \bar{Y}$ on $X - \bar{X}$

NEELS data

b) Y on \bar{X}, X

effect of group membership on individual

"contextual effect"

$$\beta_{Y\bar{X} \cdot X} = \beta_{Y\bar{X}}^b - \beta_{YX}^{w-p}$$

? how would have student have done it in different group?
within-pooled

$$\beta_{YX \cdot \bar{X}} = \beta_{YX}^{w-p} = \beta_{(Y-\bar{Y})(X-\bar{X})}$$

NEELS data

c) Y on $X, X - \bar{X}$

$$\beta_{YX \cdot (X-\bar{X})} = \beta_{YX}^b$$

$$\beta_{Y(X-\bar{X}) \cdot X} = \beta_{YX}^{w-p} - \beta_{Y\bar{X}}^b$$

some relations, adjusted variables interpretation

note: $X \cdot \bar{X} = X - \frac{\text{Cov}(X, \bar{X})}{\text{Var}(\bar{X})} \bar{X} = X - \bar{X}$ SEE REVERSE

so in (b) $\beta_{YX \cdot \bar{X}} = \beta_{Y(X-\bar{X})} = \beta_{Y(X-\bar{X})}$ relative standing

in (a) also $(X-\bar{X}) \cdot \bar{X} = X - \bar{X}$ $\beta_{Y(X-\bar{X}) \cdot \bar{X}} = \beta_{YX}^{w-p}$

more ~~comp~~ of the same,

$\bar{X} \cdot X$ residuals for \bar{X} on X regression

SEE REVERSE

$\bar{X} \cdot X = \bar{X} - \frac{\text{Cov}(X, \bar{X})}{\text{Var}(\bar{X})} X = \bar{X} - n_x^2 X$ in (b) $\beta_{Y\bar{X} \cdot X} = \beta_{Y(\bar{X} \cdot X)}$ etc

b) $\beta_{YX \cdot \bar{X}}$ coeff of X in \bar{Y}, X regression (b)

$\stackrel{\text{week 1}}{=} \beta_{Y(X \cdot \bar{X})}$

what is $X \cdot \bar{X}$? residuals from predicting X from \bar{X}

logically that's $X - \bar{X}$ w/in group

brote force in handout

$$X \cdot \bar{X} = X - \frac{\text{Cov}(X, \bar{X})}{\text{Var}(\bar{X})} \bar{X} = X - \bar{X}$$

residual predicting X from \bar{X}

slope X on $\bar{X} = 1$

$$\beta_{Y(X \cdot \bar{X})} = \frac{\text{Cov}(\bar{Y} + (Y - \bar{Y}), X - \bar{X})}{\text{Var}(X - \bar{X})}$$

$$= \frac{\text{Cov}(Y - \bar{Y}, X - \bar{X})}{\text{Var}(X - \bar{X})}$$

$$= \beta_{Y - \bar{Y}, X - \bar{X}} = \beta_{YX}^{w-p}$$