

# Math notes: Random Effects Models

Th Week 4 (II) (lme, HLM, SAS Proc Mixed)

Stat 209  
3/2/12

1. Berk text story (2-level)

score<sub>ij</sub> = student<sub>j</sub> score in school i

income<sub>ij</sub> = " income in school i

ratio<sub>i</sub> = school student-teacher ratio

level  
student  
student  
school

These "multilevel" models decompose  $\beta_{wpx}$  slopes-as-outcomes

Level 1. within school score<sub>ij</sub> =  $\eta_{0i} + \eta_{1i} \text{Income}_{ij} + e_{ij}$

Level 2 between school  $\eta_{0i} = \gamma_{00} + u_{0i}$

$\eta_{1i} = \gamma_{10} + \gamma_{11} \text{ratio}_i + u_{1i}$

no systematic link w/ ratio  
within school slope incr. w/ ratio

Intercept  $\eta_{0i}$  (level, mean intercept) w/in school slope  $\eta_{1i}$  random param vary over schools  
Ave( $\eta_{1i}$ ) is  $\beta_{wpx}$  slope

Do params of Level 1 model differ systematically over schools?

Combined model: score<sub>ij</sub> =  $\gamma_{00} + \gamma_{10} \text{Income}_{ij} + \gamma_{11} \text{ratio}_i \times \text{Inc}_{ij}$   
(fit by lme, SAS) or SFYS approach (lmList) reverse, contextual effects +  $[(\text{Inc}_{ij} \times u_{1i}) + u_{0i} + e_{ij}]$   
product term  
combine of errors

H SB model and results (Lab 2 B-R book, Singer)

Level 1 math<sub>ij</sub> =  $\alpha_{0i} + \alpha_{1i} \text{CSES}_{ij} + \epsilon_{ij}$

cses  $\bar{x}$   
so  $\alpha_0$  mean

Level 2 int  $\alpha_{0i} = \gamma_{00} + \gamma_{01} \text{meanses}_i + \gamma_{02} \text{sector}_i + u_{0i}$

slope  $\alpha_{1i} = \gamma_{10} + \gamma_{11} \text{meanses}_i + \gamma_{12} \text{sector}_i + u_{1i}$

or lme4 from lme4

```
> bryklme = lme(mathach ~ meanses*cses + sector*cses,
  random = ~ cses|school, data = Bryk)
> summary(bryklme) Linear mixed-effects model fit by REML
Random effects:
Formula: ~cses | school
StdDev Corr
(Intercept) 1.5426150 (Intr)
cses 0.3182015 0.391
Residual 6.0597955
```

Fixed effects: mathach ~ meanses \* cses + sector \* cses

|   | Value     | Std. Error | DF   | t-value  | p-value |
|---|-----------|------------|------|----------|---------|
| (Intercept) $\hat{\gamma}_{00}$         | 12.127931 | 0.1992919  | 7022 | 60.85510 | 0e+00   |
| meanses $\hat{\gamma}_{01}$             | 5.332875  | 0.3691684  | 157  | 14.44564 | 0e+00   |
| cses $\hat{\gamma}_{10}$                | 2.945041  | 0.1556005  | 7022 | 18.92694 | 0e+00   |
| sectorCatholic $\hat{\gamma}_{02}$      | 1.226579  | 0.3062733  | 157  | 4.00485  | 1e-04   |
| meanses:cses $\hat{\gamma}_{11}$        | 1.039230  | 0.2988971  | 7022 | 3.47688  | 5e-04   |
| cses:sectorCatholic $\hat{\gamma}_{12}$ | -1.642674 | 0.2397800  | 7022 | -6.85076 | 0e+00   |

Number of Observations: 7185 Number of Groups: 160

variance components error terms

Lab 2 lme fits combined model in lab text (substitute L2  $\rightarrow$  L1)

Week 5 ancova equivalence

Cath schools higher mean, more egalitarian  
[group level ancova (non-equiv groups week 5)]

Nonlinear, ex Logistic Level 1 model

a. Clinical trial (drug, control) cure oil w/in 8 clinics NLMixed  
b. Thailand repeat oil outcome Level 1 sex Level school SPS for L1 intercept  
Week 9 Growth Curves

Week 4 Random effects

② WEEK 4 TH

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# Multiple Regression v. Random Effects

## Models for (old-fashioned) Contextual Effects

OLS regression, data  
multilevel

$$Y_{ij} = \beta_0 + \beta_1 \bar{X}_{i.} + \beta_2 X_{ij} + \epsilon_{ij}$$

$$\beta_1 = \beta_{Y \bar{X} \cdot X} = \beta_{Y \bar{X}}^0 - \beta_{Y X}^{w-p}$$

effect of group on individual  
multiregr interpretation  
"increase in Y for increase X with X constant"?  
As if by experiment?

NEELS data  $\hat{\beta}^0 = 3.6$   $\hat{\beta}^b = 7$   
 $\hat{\beta}^{w-p} = 2.1$

or delusional?

### Part II

Refer to the two-level random-effects model story, taken from the Berk text, on class handout 1/31.

In the Level-1 model the outcome is student test score, and within-school predictor is student's family income.

Can you construct a level-2 model such that the resulting combined model (i.e. what would be fit by lme) is the contextual effects regression model for student test score and student income [see class handouts 1/29 and again 1/31 for the form of the contextual effects regression]

Under this multilevel model, what is the interpretation of the contextual effect?

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problem

vars

$$Y_{ij} = \text{score}_{ij}$$

$$X_{ij} = \text{income}_{ij}$$

### Solution?

Level I

$$Y_{ij} = \eta_{0i} + \eta_{1i} X_{ij} + u_{ij}$$

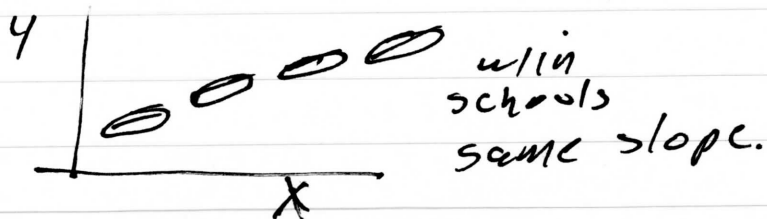
Level II

$$\eta_{0i} = \gamma_0 + \gamma_1 \bar{X}_{i.} \quad (\text{deterministic})$$

$$\eta_{1i} = \beta^{w-p} \quad (\text{all schools, same slope})$$

$$Y_{ij} = \gamma_0 + \gamma_1 \bar{X}_{i.} + \beta^{w-p} X_{ij} + u_{ij}$$

"ANCOVA" model Kruskal-Wallis test



random effects model

combined model

delusional also?  $\gamma_1 = \beta_{Y \bar{X} \cdot X}$   
effect group level in outcome