PROBLEM 4   Belson estimate

In Week 5 (see handout) and HW5 prob 7 we considered the Belson estimate
for observational studies
Belson, W. A. (1956), “A technique for studying the effects of a television broadcast,”
Applied Statistics, 5, 195–202.

AKA Peters-Belson, this estimate is widely used at present in health outcomes research and
in wage-discrimination studies.
A representative statement (Medical Care Volume 42, Number 8, August 2004)

"The PB approach has been used in wage discrimination studies and race (sex)
discrimination cases13 to predict the experience a minority (female) individual would have
had if they were white (male). The conventional regression approach, which includes a
dummy variable to identify race/ ethnicity, assumes a common amount (degree) of disparity
for all minority group members regardless of their individual characteristics. In
contrast, the PB method produces estimates of disparity for each minority group member by
incorporating their individual characteristics. Our study explores how the PB approach can
be similarly used to understand disparities in public health outcomes as illustrated from
studying cancer screening."

model $Y = \beta_1 + \beta_2 G + \beta_3 X + \beta_4 XG + \varepsilon$

Grp 1 $E(Y|X, G=1) = \beta_1 + \beta_2 + (\beta_3 + \beta_4)X$

Grp 0 $E(Y|X, G=0) = \beta_1 + \beta_3 X$

Treatment effect (diff of regressions) $\Delta(X) = \beta_2 + \beta_4 X$ func of X iff $\beta_4 \neq 0$

abscissa of point of intersection (w/in group regression) "cut-off" $X^0 = -\beta_2/\beta_4$

ATI research assignment on "aptitude" to differential instruction.

Start with

$D(\bar{X}_1)$ diff of regressions at treatment group mean on X

notate the within group slopes Y on X as: $\hat{\delta}_1$ treatment slope

$\hat{\delta}_0$ control slope

$$D(\bar{X}_1) = \hat{\beta}_2 + \hat{\beta}_4 \bar{X}_1$$

$$= (\bar{Y}_1 - \hat{\delta}_1 \bar{X}_1) - (\bar{Y}_0 - \hat{\delta}_0 \bar{X}_0) \qquad [\hat{\beta}_2 \text{ part}]$$

$$+ (\hat{\delta}_1 - \hat{\delta}_0)\bar{X}_1 \qquad [\beta_4 \bar{X}_1 \text{ part}]$$

cancel terms and regroup

$$D(\bar{X}_1) = (\bar{Y}_1 - \bar{Y}_0) - \hat{\delta}_0 (\bar{X}_1 - \bar{X}_0)$$

gives form of Belson estimator! use control group slope in ancova-style adjustment.

HW5 Belson Stat 209 week 5

refer to comparing regressions handout CNRL Math notes

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start with $D(\bar{X}_1)$ diff of regressions at treatment group mean on X

for easy notation make:

$\hat{\delta}_1$ the Y on X slope in treatment group

$\hat{\delta}_0$ the Y on X slope in control group

[in CNRL model in handout $\hat{\delta}_0 = \hat{\beta}_3$, $\hat{\delta}_1 = \hat{\beta}_3 + \hat{\beta}_4$]

$$D(\bar{X}_1) = \hat{\beta}_2 + \hat{\beta}_4 \bar{X}_1$$ from CNRL handout

$$= (\bar{Y}_1 - \hat{\delta}_1 \bar{X}_1) - (\bar{Y}_0 - \hat{\delta}_0 \bar{X}_0)$$ $\hat{\beta}_2$ part

$$+ (\hat{\delta}_1 - \hat{\delta}_0)\bar{X}_1$$ $\hat{\beta}_4$ part

cancel terms, regroup

$$D(\bar{X}_1) = (\bar{Y}_1 - \bar{Y}_0) - \hat{\delta}_0 (\bar{X}_1 - \bar{X}_0)$$

as indicated in class handout use control group slope in ancova style adjustment.

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small note of $\hat{\beta}_2$ part above

for OLS fit $\bar{Y} = a + b\bar{X}$ so $a = \bar{Y} - b\bar{X}$ form of control and treatment groups above.