ANCOVA, CNRL equations

precursor: t-test $y = \beta_0 + \beta_1 G + \varepsilon$ $\quad \hat{\beta}_1/se(\hat{\beta}_1)$

pooled t-test (not Welch)

$G = 0, 1$
group membership

$$y = \gamma_0 + \gamma_1 G + \gamma_2 X + \varepsilon$$

link w/ data (A)

ancova

$$\hat{\gamma}_1 = \bar{Y}_1 - \bar{Y}_0 - \hat{\delta}_p (\bar{X}_1 - \bar{X}_0)$$

constant treatment effect

$$\hat{\gamma}_2 = \hat{\delta}_p \quad \text{ave w/in group slopes}$$

cf. overheads   deriv on board   HSB school·level ex

more general model (CNRL)

(B) $$y = \beta_1 + \beta_2 G + \beta_3 X + \beta_4 XG + \varepsilon$$

see Berk taxa ATI in tros

"interaction" term

diff regression lines (cf Berk) w/in group
$\hat{\beta}_2$, careful, diff of lines at $X = 0$

$$\Delta(X) = \beta_2 + \beta_4 X \qquad \text{treatment effect as function of X}$$

(C) $$D(X) = \hat{\beta}_2 + \hat{\beta}_4 X \qquad \text{sample estimate}$$

$\hat{cov}(\hat{\beta}_i, \hat{\beta}_j)$: s_{ij}

sampling variance $\qquad s^2_{D(X)} = s^2_{D(C_a)} + s_{44}(X - C_a)^2$ (G)

center of accuracy (D) $\quad C_a = \dfrac{-s_{24}}{s_{44}}$

min
$D(C_a) = $ ancova (E) $\hat{\gamma}_1$; $\quad s^2_{D(C_a)} = s_{22} + s_{24} C_a$

$D(X)/s_{D(X)}$ pick-a-point

Regions of significance, J-N

(H) $D(X)$ "significantly" different from 0
simultaneous R' $\qquad 2 F_{2,n-4}$
cf CNRL paper. p.318 R' $\qquad$ working-hotelling

CNRL Math Notes

model $Y = \beta_1 + \beta_2 G + \beta_3 X + \beta_4 XG + \varepsilon$

Grp 1 $E(Y|X, G=1) = \beta_1 + \beta_2 + (\beta_3 + \beta_4)X$

Grp 0 $E(Y|X, G=0) = \beta_1 + \beta_3 X$

Treatment effect $\Delta(X) = \beta_2 + \beta_4 X$ func of X iff
(diff of regressions) $\beta_4 \neq 0$

abscissa of point
of intersection $X^0 = -\beta_2/\beta_4$ ATI research
(w/in group regression)   assignment on
"cut-off"   "aptitude" to
differential
instruction.

Inference

From sample obtain estimates $\hat{\beta}_i$ $i = 1, \ldots, 4$

$D(X) = \hat{\beta}_2 + \hat{\beta}_4 X$ S_{ij}, elements of $\hat{Cov}(\hat{\beta}_i)$
4×4

$\hat{X}^0 = -\hat{\beta}_2/\hat{\beta}_4$ ratio estimator (biased)

inference for $\Delta(X)$ pick-a-point
$D(C_a)$ ancova treatment effect; $D(\bar{X}_G)$ average treatment effect
$D(\bar{X}_{sub})$ average treatment effect
for subgroup

sampling variance $S_{D(X)}^2 = S_{22} + S_{24} C_a + S_{44}(X - C_a)^2$, where $C_a = -S_{24}/S_{44}$

$\Delta(X)$ inference: $D(X)/S_{D(X)}$ via t-distrib N-4 df, usual CI

Inference for $\Delta(X)$ J-N region of significance

X-values s.t. reject $\Delta(X) = 0$

R non-simultaneous $D(X) \pm \sqrt{F_{1, N-4}^{\alpha} S_{D(X)}^2}$ concatenate $D(X)/S_{D(X)}$
tests or CI

R' simultaneous   proper   Working-Hotelling band about D(X)

hyperbolas $D(X) \pm \sqrt{2 F_{2, N-4}^{\alpha} S_{D(X)}^2}$ R' {values on X-axis outside WH band}

$(R > R')$ Potthoff 1966