

Instrumental Variable Methods

STAT 266
week 9

Observational Studies

Omitted Variables: Fixing Broken Regressions

$$Y = \beta_0 + \beta_1 D + u$$

Dose-response, (returns to schooling)
 $Y = \text{wage}$ $D = \text{educ}$ but $\text{Cov}(D, u) \neq 0$

$$Y = \beta_0 + \beta_1 G + u$$

Group membership effects, t-test

"Broken" \rightarrow D, G correlated with omitted variables in u

but $\text{Cov}(G, u) \neq 0$

OLS fails for $Y = \beta_0 + \beta_1 X + u$ when $\text{Cov}(X, u) \neq 0$
e.g. $\log(\text{wage}) = \beta_0 + \beta_1 X + u$ (ability omitted, Angrist)

To the rescue? instrument Z such that $\text{Cov}(X, Z) \neq 0$

AND $\text{Cov}(Z, u) = 0$

empirical assoc. strong, weak instr
if true ancova also works

cue Dusty's. hope, untestable

Z "exogenous" no partial effect on Y (even if Z random ass in RCT)

Properties: $Y = \beta_0 + \beta_1 X + u \Rightarrow$

$$\text{Cov}(Z, Y) = \beta_1 \text{Cov}(Z, X) + \text{Cov}(Z, u)$$

thus $\beta_1^{IV} = \text{Cov}(Z, Y) / \text{Cov}(Z, X)$ (Z replaces X)

$$\hat{\beta}_1^{IV} = \frac{S_{YZ}}{S_{XZ}} \quad \hat{\beta}_0^{IV} = \bar{Y} - \hat{\beta}_1^{IV} \bar{X}, \quad \text{Var}(\hat{\beta}_1^{IV}) = \frac{\hat{\sigma}^2}{S_{XZ} \cdot r_{XZ}^2}$$

by wish upon a star
weak instrument?
 $(r_{XZ} = 1 \Rightarrow n/100)$

RCT IV Random Assignment $G = 1, 0$

Encouragement Designs

Compliance Adjustments

Clever, innovative designs for estimating Dose-Response

Dose, binary or measured

$G = 1, 0$ encourage or not (RCT)

D : self-selected dose

Y outcome

IV assume assignment G no effect on Y

$$\hat{\beta}_{IV} = \frac{S_{YG}}{S_{DG}} = \frac{\bar{Y}_1 - \bar{Y}_0}{\bar{D}_1 - \bar{D}_0}$$

Wald estimator

salt, sesame sb. in session

Desperate attempt to adjust, salvage broken protocols in RCT
Compliance seldom binary

Binary Compliance c, n
no crossover $\pi_c = P(T|G=1)$

ITT $\mu_1 - \mu_0$

CACE $\mu_{c1} - \mu_{c0} = \frac{ITT}{\pi_c}$ iff

$\mu_{n1} - \mu_{n0} = 0$ ER, IV assumption

$\mu_1 - \mu_0 = \pi_c (\mu_{c1} - \mu_{c0}) +$

$(1 - \pi_c) (\mu_{n1} - \mu_{n0}) \rightarrow 0$
examples in session

A. Observational data (adapted from Lab 3 Stat209) Mroz data, returns to schooling
 ##### Examples from Wooldridge, Introductory Econometrics ##### Chapters 15 and 16
 ## stata results available from
<http://fmwww.bc.edu/gstat/examples/wooldridge/wooldridge15.html>

```
> lm.posWage = lm(logWage ~ educ) > summary(lm.posWage)#session has full output
Coefficients: #year increase in educ, fit increases 11 cents hourly wage
              Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.1852      0.1852  -1.000   0.318
educ         0.1086      0.0144   7.545 2.76e-13 ***
Multiple R-Squared: 0.1179, Adjusted R-squared: 0.1158
F-statistic: 56.93 on 1 and 426 DF, p-value: 2.761e-13
> # we get a highly significant slope (but not big Rsq),
```

```
> install.packages("AER") > library(AER) > help(ivreg)
> ivreg1 = ivreg(logWage ~ educ | fatheduc)
##### use the diagnostics option of ivreg (recent)
> summary(ivreg1, diagnostics = TRUE)
Call: ivreg(formula = logWage ~ educ | fatheduc)
Coefficients: # matches tsls Task1
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.44110      0.44610   0.989  0.3233
educ         0.05917      0.03514   1.684  0.0929 .
```

Diagnostic tests:

	df1	df2	statistic	p-value
Weak instruments	1	426	88.84	<2e-16 ***
Wu-Hausman	1	425	2.47	0.117
Sargan	0	NA	NA	NA

```
---
Residual standard error: 0.6894 on 426 degrees of freedom
Multiple R-Squared: 0.09344, Adjusted R-squared: 0.09131
Wald test: 2.835 on 1 and 426 DF, p-value: 0.09294
> confint(ivreg1) # parameter confidence intervals as usual
              2.5 %      97.5 %
(Intercept) -0.433239996 1.3154468
educ         -0.009703131 0.1280501
```

Investigate the 'Weak instruments' entry, matches cor.test

```
> cor(educ, fatheduc)
```

```
[1] 0.415403
```

just a test (like in the main lab of correlation between predictor and instrument

```
# the Hausman test is often used to show there's a difference between OLS and IV,
# but what does that tell you?
```

```
# ivreg diagnostics finds discrepancy non-significance even though IV est cuts the OLS
# estimate in half and IV estimate now non-significant
```

```
## to see the details of the Hausman test (and do it by hand) see pdf page 46 onward from
http://personal.rhul.ac.uk/uhte/006/ec2203/Lecture%2015\_IVestimation.pdf
also the Basel class notes linked in week 6
```

```
## The Sargan test is a statistical test used for testing over-identifying restrictions
## relevant to simultaneous eqs, which we investigated in identifiability in lab script
```

```
# some package ivmodel display in week9 RQ
```

```
#####
in Rogosa session B. Encouragement Design C. Compliance Adjustments
```