## Advanced Statistical Methods for Observational Studies

LECTURE 01

## introduction

this class

## this class

- Website


## this class

- Website
- Expectations


## this class

- Website
- Expectations
- Questions


## observational studies

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We'll almost exclusively use matching to implement the different study designs we introduce.

There are other, entirely reasonable, ways to implement these study designs.

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## a small bit about me

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- I do causal inference


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- I do causal inference:
- Observational studies of:


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- I do causal inference:
- Observational studies of: cardiothoracic interventions


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- I do causal inference:
- Observational studies of: cardiothoracic interventions, neonates


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- I do causal inference:
- Observational studies of: cardiothoracic interventions, neonates, and criminology.


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- I do causal inference:
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- Randomized studies: six interventions here at Stanford to improve educational outcomes, two large trials of a sexual assault prevention program.


## an aside...

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- You can call me "Mike"


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- If you want to use my last name, Baiocchi, totally feel free to...


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- It feels hard.
- Because many of us don't really know much about the real world...


## design

## RANDOMIZATION AND SAMPLING

## where does the data come from?

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- We design trials.


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- Understand population from subsets of those populations
- Both use elements of control and randomness


## an example: randomization

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- Want to study a pill.


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- Want to study a pill.
- Design the study


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- Want to study a pill.
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- Uniform randomization


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- Want to study a pill.
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- Matched pairs randomization


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David A. Freedman

## where data come from

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## CAUSALITY



## JUDEA PEARL

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## dis•mal sci•ence

noun humorous
economics.

## Translations, word origin, and more definitions

The dismal science - Wikipedia, the free encyclopedia
https://en wikipedia.org/wiki/The_dismal_science • Wikipedia *
"The dismal science" is a derogatory alternative name for economics coined by the Victorian historian Thomas Carlyle in the 19th century. The term drew a contrast with the then-familiar use of the phrase "gay science" to refer to song and verse writing. Origin - Criticism - Beyond Carlyle - See also
inference

## picking inference

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- Linear regression:


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- Known assignment mechanism to T or C
- "Fancier" methods tend to have more assumptions... and thus leave you open to more lines of attack.
- These attacks can be obviated by careful preparation during the design phase.


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- If you want to accomplish more, collect more data or do additional analyses. ("If have to use something more complicated than a t-test then someone messed up...")
- The fewer assumptions there are, the easier it will be to perform a "sensitivity analysis" - build an argument to beat back the haters.


## picking inference

- Another option:


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This paper presents a breakthrough in rhetorical logic, a promising field of science, of great values to those writing research proposals. It provides new, and utterly convincing tools for closing embarrassing gaps in your reasoning, without having to resort to "brute-force" methods such as actually thinking about the problem in the first place.

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- Another option: Proof by intimidation

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- Michael Wilkinson (Annals of Improbable Research 2000)


## observational study design

## Application: Regionalization

- Hospitals vary in their ability to care for premature infants.
- The American Academy of Pediatrics recognizes levels: 1, 2, 3A, 3B, 3C, 3D and Regional Centers.
- Regionalization of care refers to a policy that suggests or requires that high-risk mothers deliver at hospitals with greater levels of capabilities.




$2 \%$



## Outcome

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$\uparrow$

## The data

- Every baby delivered in a $10+$ year period
- California
- Pennsylvania
- Missouri
- Mothers' information
- ICD9 codes
r Delivery
- Post-delivery complications
- Some pre-delivery
- Some SES information
- Zip code of residence
- Birth/death certificates
- Census information
- PA and MO have zip code level
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$\uparrow$

## Summary of Problem

- Want to quantify effect of level of NICU on rate of death
- Observational data
- Sorting bias
- Some sorting variables are unobserved









## a matched study

## outline of a study

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Introduction

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|  | Variable Type | High NICU | Low NICU | sd | $\Delta / \mathrm{sd}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Mortality | Outcome | 2.26\% | 1.25\% | 13.33\% | 0.08 |
| Difference in Travel Time | Instrument | 4.57 | 19.00 | 17.18 | -0.84 |
| \% attending high level NICU | Treatment | 100.0\% | 0.0\% | 49.7\% | 2.01 |
| Birth weight | Preemie covariates | 2,454.07 | 2,693.24 | 739.27 | -0.32 |
| Gestational age |  | 34.61 | 35.69 | 2.80 | -0.39 |
| GI | $\%$ of preemies with type of congenital disorders | 0.9\% | 0.6\% | 8.7\% | 0.04 |
| GU |  | 0.9\% | 0.8\% | 9.0\% | 0.01 |
| CNS |  | 0.9\% | 0.4\% | 8.3\% | 0.05 |
| Pulmonary |  | 0.8\% | 0.7\% | 8.8\% | 0.01 |
| Cardio |  | 1.4\% | 0.7\% | 10.5\% | 0.06 |
| Skeletal |  | 0.7\% | 0.9\% | 9.0\% | -0.02 |
| Skin |  | 0.0\% | 0.0\% | 0.0\% | 0.00 |
| Chromosomes |  | 0.4\% | 0.3\% | 6.3\% | 0.02 |
| Other_Anomaly |  | 0.8\% | 0.1\% | 7.0\% | 0.09 |
| Gestational_DiabetesM | Mother covariates | 4.9\% | 4.3\% | 21.0\% | 0.03 |
| Mother's education |  | 3.76 | 3.58 | 1.19 | 0.16 |
| Insurance - Fee for service |  | 24.0\% | 24.5\% | 42.8\% | -0.01 |
| Insurance - HMO |  | 32.3\% | 27.8\% | 46.0\% | 0.10 |
| Insurance - Government |  | 23.5\% | 24.2\% | 42.6\% | -0.02 |
| Insurance - Other |  | 16.8\% | 21.4\% | 39.1\% | -0.12 |
| Uninsured |  | 2.2\% | 1.6\% | 13.7\% | 0.04 |
| Prenatal care |  | 2.51 | 2.37 | 1.30 | 0.11 |
| Single birth (y/n) |  | 79.0\% | 86.1\% | 38.3\% | -0.18 |
| Parity |  | 2.08 | 2.09 | 1.31 | -0.01 |
| Mother's age |  | 28.41 | 27.71 | 6.25 | 0.11 |
| Median income | Census level covariates | 41,484.25 | 40,258.92 | 14,587.24 | 0.08 |
| Median home value |  | 97,663.00 | 95,083.15 | 48,762.43 | 0.05 |
| \% completed high school |  | 79.9\% | 80.0\% | 9.7\% | -0.01 |
| \% completed college |  | 22.2\% | 19.4\% | 13.1\% | 0.21 |
| \% renting |  | 31.4\% | 27.9\% | 12.8\% | 0.28 |
| \% below poverty line |  | 13.4\% | 11.8\% | 9.9\% | 0.16 |

the data

## the data

Unadjusted comparison of T vs C

|  | High NICU | Low NICU | sd | $\Delta /$ sd |
| :--- | ---: | ---: | ---: | ---: |
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| death | $1.55 \%$ | $1.94 \%$ | $13.67 \%$ | -0.03 |
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## matching two observations

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## matching two observations

- Exact matching.
- Exact matching would be awesome, but consider how unlikely it is to be achievable.
- Wonderful to have some summary of how "close" two observations are to each other.


## the propensity score



## the propensity score



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## the propensity score



$$
e(x)=
$$

## the propensity score



$$
e(\boldsymbol{x})=\operatorname{Pr}(d=1 \mid \boldsymbol{x})
$$

## the propensity score


$e(\boldsymbol{x})=\operatorname{Pr}(d=1 \mid \boldsymbol{x})=f($ birth weight, gestational age, mother's insurance,$\ldots)$

## the propensity score



$$
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$$

## the propensity score


$e(\boldsymbol{x})=\operatorname{Pr}(d=1 \mid \boldsymbol{x})=f(\boldsymbol{x})=$ feed in a high dimensional vector that describes the observations at baseline; get out a one-dimensional summary

## propensity score

- Exact matching
- Exact matching would be awesome, but consider how unlikely it is to be achievable.
- Wonderful to have some summary of how "close" two observations are to each other.


## results: "table 1"

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Unadjusted comparison of T vs C

|  |  | High NICU | Low NICU | sd | $\Delta /$ sd |
| :--- | :--- | ---: | ---: | ---: | ---: |
|  | n $=180,000$ | $2.26 \%$ | $1.25 \%$ | $13.67 \%$ | 0.07 |
|  | death | 2,454 | 2,693 | 739 | -0.32 |
|  | birth weight (g) | 34.61 | 35.69 | 2.76 | -0.39 |

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Matched comparison of T vs C

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| $\mathrm{n}=120,000$ | $1.55 \%$ | $1.94 \%$ | $13.67 \%$ | -0.03 |  |
|  | death | 2,584 | 2,581 | 739 | 0.00 |
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Unlike a Table 1 from an RCT, we aren't verifying that a randomization seems to have worked. Instead, we are heading off a "you obviously stacked the deck" argument.

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- If the reader is willing to accept this then we move on to the analysis of the groups.
- Inference can be done in many different forms (largely driven by your upbringing). The use of matching in the study design phase meshes well with randomization test type inference.


## takeaways

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- Matching on observables is possible (deatils next)


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- The naïve model is our foundation for doing these forms of analyses
- Easier to build a sensitivity model


## basic tools

## PAIR MATCHING

tools
(0)

## tools

- Propensity scores


## tools

- Propensity scores
- Distance matrices


## tools

- Propensity scores
- Distance matrices
- Calipers and penalty functions


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## example: NICU

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- If you have 40 binary covariates then you have $2^{40} \cong 1.1 * 10^{12}$.
- Continuous variables make exact matching even harder.
- We quickly get into questions about "close enough."
- We also get into the idea that not all covariates are equally important in determining which observations are "similar."


## propensity scores

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- Uniform randomization has $1 / 2$ by construction


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## propensity scores

- Strongly Ignorable Treatment Assignment:


## propensity scores

- Strongly Ignorable Treatment Assignment: Those that look alike (in our data set) are alike


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- Strongly Ignorable Treatment Assignment: Those that look alike (in our data set) are alike

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\pi_{i}=\operatorname{Pr}\left(Z_{i}=1 \mid r_{T i}, r_{C i}, \boldsymbol{x}_{i}, u_{i}\right)=\operatorname{Pr}\left(Z_{i}=1 \mid \boldsymbol{x}_{i}\right)
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- Therefore the mismatches in $\boldsymbol{x}$ will be due to chance and will tend to balance.


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- Histograms of propensity scores
- "Table 1 " (discussed below)


## propensity scores

- How does one estimate the propensity score


## propensity scores

- How does one estimate the propensity score

| obs | b_weight | gest_age | dose | death |
| ---: | ---: | ---: | ---: | ---: |
| 1 | 2412 | 36 | 1 | 0 |
| 2 | 2205 | 29 | 1 | 1 |
| 3 | 2569 | 36 | 1 | 0 |
| 4 | 2443 | 34 | 1 | 0 |
| 5 | 2569 | 36 | 0 | 0 |
| 6 | 2436 | 35 | 0 | 0 |
| 7 | 2461 | 34 | 0 | 0 |
| 8 | 2759 | 32 | 0 | 0 |
| 9 | 2324 | 27 | 0 | 1 |
| 10 | 2667 | 34 | 0 | 0 |

## propensity scores

- How does one estimate the propensity score: logistic model

| obs | b_weight | gest_age | dose | death |
| ---: | ---: | ---: | ---: | ---: |
| 1 | 2412 | 36 | 1 | 0 |
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## propensity scores

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$$
\operatorname{logit}(\text { dose })=\beta_{0}+\beta_{1} * \mathrm{~b} \_ \text {weight }+\beta_{2} * \text { gest_age }
$$

| obs | b_weight | gest_age | dose | death |
| ---: | ---: | ---: | ---: | ---: |
| 1 | 2412 | 36 | 1 | 0 |
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| obs | b_weight | gest_age | dose | death | $\mathbf{e}^{\wedge}(\mathbf{x})$ |
| ---: | ---: | ---: | ---: | ---: | ---: |
| 1 | 2412 | 36 | 1 | 0 |  |
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| obs | b_weight | gest_age | dose | death | $\mathbf{e}^{\boldsymbol{n}(x)}$ |
| ---: | ---: | ---: | ---: | ---: | ---: |
| 1 | 2412 | 36 | 1 | 0 | 0.54 |
| 2 | 2205 | 29 | 1 | 1 | 0.43 |
| 3 | 2569 | 36 | 1 | 0 | 0.57 |
| 4 | 2443 | 34 | 1 | 0 | 0.53 |
| 5 | 2569 | 36 | 0 | 0 | 0.57 |
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| 6 | 2436 | 35 | 0 | 0 | 0.54 |
| 7 | 2461 | 34 | 0 | 0 | 0.53 |
| 8 | 2759 | 32 | 0 | 0 | 0.57 |
| 9 | 2324 | 27 | 0 | 1 | 0.43 |
| 10 | 2667 | 34 | 0 | 0 | 0.57 |

## propensity scores

- How does one estimate the propensity score: logistic model

$$
\operatorname{logit}(\widehat{\operatorname{dose}})=\hat{\beta}_{0}+\hat{\beta}_{1} * \text { b_weight }+\hat{\beta}_{2} * \text { gest } \_ \text {age }
$$

| obs | b_weight | gest_age | dose | death | $\mathbf{e}^{\boldsymbol{\wedge}(\mathbf{x})}$ |
| ---: | ---: | ---: | ---: | ---: | ---: |
| 1 | 2412 | 36 | 1 | 0 | 0.54 |
| 2 | 2205 | 29 | 1 | 1 | 0.43 |
| 3 | 2569 | 36 | 1 | 0 | 0.57 |
| 4 | 2443 | 34 | 1 | 0 | 0.53 |
| 5 | 2569 | 36 | 0 | 0 | 0.57 |
| 6 | 2436 | 35 | 0 | 0 | 0.54 |
| 7 | 2461 | 34 | 0 | 0 | 0.53 |
| 8 | 2759 | 32 | 0 | 0 | 0.57 |
| 9 | 2324 | 27 | 0 | 1 | 0.43 |
| 10 | 2667 | 34 | 0 | 0 | 0.57 |

$\operatorname{logit}(\widehat{\text { dose }})=-0.3+0.0002 *$ b_weight $+0.01 *$ gest_age

## propensity scores

- Why logistic?


## propensity scores

- Why logistic?
- I've got two answers:
(i)
(ii)


## propensity scores

- Why logistic?
- I've got two answers:
(i) It's what we do. [cultural]
(ii)


## propensity scores

-Why logistic?

- I've got two answers:
(i) It's what we do. [cultural]
(ii) Consider how a regression tree might produce different results. [technical]


## propensity scores

- Question: if you knew the propensity score would you want to use it in lieu of the estimated propensity score?


## propensity scores

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## propensity scores

- Question: if you knew the propensity score would you want to use it in lieu of the estimated propensity score?
- There are (at least) two valid answers: (i) no, for balance purposes ( 1 and 2 ) and (ii) yes, because of invalid inference (1 and 2 ).


## takeaways: propensity scores

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- Two key features:
- Propensity: used for inference
- Score: used for creating two groups


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## takeaways: propensity scores

- Two key features:
- Propensity: used for inference
- Score: used for creating two groups
- Dimensional reduction technique:
- Not guaranteed to match two people who look alike
- Histograms of covariates
- Histograms of propensity scores
- Reasons to use propensity score:
(i) matching on $e(\boldsymbol{x})$ is often practical even when there are many covariates in $\boldsymbol{x}$ because $e(\boldsymbol{x})$ is a single variable,
(ii) matching on $e(\boldsymbol{x})$ tends to balance all of $\boldsymbol{x}$, and
(iii) failure to balance $e(\boldsymbol{x})$ implies that $\boldsymbol{x}$ is not balanced.


## distance matrices

distance matrices

## distance matrices

- How do we summarize?


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- If you're matching treated to control, then it's a matrix with:


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- Each entry in the matrix represents a "distance" between a treated and control unit


## distance matrices

- How do we summarize?
- If you're matching treated to control, then it's a matrix with:
- A row for each treated
- A column for each control
- Each entry in the matrix represents a "distance" between a treated and control unit
- We sum up the entries of those that are matched to get an overall metric of quality of match. The algorithms are targeted toward minimizing these sums.


## distance matrices

| obs | $\mathbf{e}^{\wedge}(\mathbf{x})$ |
| ---: | ---: |
| 1 | 0.54 |
| 2 | 0.43 |
| 3 | 0.57 |
| 4 | 0.53 |
| 5 | 0.57 |
| 6 | 0.54 |
| 7 | 0.53 |
| 8 | 0.57 |
| 9 | 0.43 |
| 10 | 0.57 |

## distance matrices

| obs | $e^{\wedge}(x)$ |
| ---: | ---: |
| 1 | 0.54 |
| 2 | 0.43 |
| 3 | 0.57 |
| 4 | 0.53 |
| 5 | 0.57 |
| 6 | 0.54 |
| 7 | 0.53 |
| 8 | 0.57 |
| 9 | 0.43 |
| 10 | 0.57 |


|  | 5 | 6 |
| :---: | :---: | :---: |
| 1 | $/$ |  |
|  |  |  |
| 2 |  |  |
| 3 |  |  |
|  | ( |  |
| 4 | ( |  |

$8 \quad 9 \quad 10$


## distance matrices

| obs | $\mathrm{e}^{\wedge}(\mathrm{x})$ |  | 5 | 6 | 7 | 8 | 9 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0.54 |  | $/$ |  |  |  |  |  |
| 2 | 0.43 | 1 |  |  |  |  |  | 人 |
| 3 | 0.57 |  |  |  |  |  |  |  |
| 4 | 0.53 | 2 |  |  |  |  |  |  |
| 5 | 0.57 |  |  |  |  |  |  |  |
| 6 | 0.54 | 3 |  |  |  |  |  |  |
| 7 | 0.53 |  | $1$ |  |  |  |  | ) |
| 8 | 0.57 |  |  |  |  |  |  | ) |
| 9 | 0.43 |  |  |  |  |  |  |  |
| 10 | 0.57 |  |  |  |  |  |  |  |

We can describe a distance in pretty much any way we want.

## distance matrices

difference: $\hat{e}\left(\boldsymbol{x}_{i}\right)-\hat{e}\left(\boldsymbol{x}_{j}\right)$

| obs | $e^{\wedge}(\mathbf{x})$ |
| ---: | ---: |
| 1 | 0.54 |
| 2 | 0.43 |
| 3 | 0.57 |
| 4 | 0.53 |
| 5 | 0.57 |
| 6 | 0.54 |
| 7 | 0.53 |
| 8 | 0.57 |
| 9 | 0.43 |
| 10 | 0.57 |

$\left.\begin{array}{l} \\ 1 \\ 2 \\ 3 \\ 4\end{array} \begin{array}{cccccc}5 & 6 & 7 & 8 & 9 & 10 \\ & & & & & \\ & & & & & \\ & & & & & \end{array}\right)$

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| obs | $\mathrm{e}^{\wedge}(\mathbf{x})$ |
| ---: | ---: |
| 1 | 0.54 |
| 2 | 0.43 |
| 3 | 0.57 |
| 4 | 0.53 |
| 5 | 0.57 |
| 6 | 0.54 |
| 7 | 0.53 |
| 8 | 0.57 |
| 9 | 0.43 |
| 10 | 0.57 |


| 5$\quad 6$ |
| :---: |
| 1 |
| 1 |
| 2 |
| 3 |
| 4 | | -0.03 | 0.00 | 0.01 | -0.03 | 0.11 | -0.03 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| -0.14 | -0.11 | -0.10 | -0.14 | 0.00 | -0.14 |
| 0.00 | 0.04 | 0.04 | 0.00 | 0.14 | 0.00 |
| -0.05 | -0.01 | 0.00 | -0.04 | 0.09 | -0.04 |

We can describe a distance in pretty much any way we want.

## distance matrices

difference: $\hat{e}\left(\boldsymbol{x}_{i}\right)-\hat{e}\left(\boldsymbol{x}_{\boldsymbol{j}}\right)$

| obs | $\mathbf{e n}^{\wedge}(\mathbf{x})$ |
| ---: | ---: |
| 1 | 0.54 |
| 2 | 0.43 |
| 3 | 0.57 |
| 4 | 0.53 |
| 5 | 0.57 |
| 6 | 0.54 |
| 7 | 0.53 |
| 8 | 0.57 |
| 9 | 0.43 |
| 10 | 0.57 |


| 5$\quad 6$ |
| :---: |
| 1 |
| 1 |
| 2 |
| 3 |
| 4 | | -0.03 | 0.00 | 0.01 | -0.03 | 0.11 | -0.03 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| -0.14 | -0.11 | -0.10 | -0.14 | 0.00 | -0.14 |
| 0.00 | 0.04 | 0.04 | 0.00 | 0.14 | 0.00 |
| -0.05 | -0.01 | 0.00 | -0.04 | 0.09 | -0.04 |

We can describe a distance in pretty much any way we want.

## distance matrices

absolute difference: $\left|\hat{e}\left(\boldsymbol{x}_{\boldsymbol{i}}\right)-\hat{e}\left(\boldsymbol{x}_{j}\right)\right|$

| obs | $e^{\wedge}(\mathbf{x})$ |
| ---: | ---: |
| 1 | 0.54 |
| 2 | 0.43 |
| 3 | 0.57 |
| 4 | 0.53 |
| 5 | 0.57 |
| 6 | 0.54 |
| 7 | 0.53 |
| 8 | 0.57 |
| 9 | 0.43 |
| 10 | 0.57 |


|  | 5 | 6 | 7 | 8 |  | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $0.03$ | 0.00 | 0.01 | 0.03 | 0.11 | 0.03 |
| 2 | 0.14 | 0.11 | 0.10 | 0.14 | 0.00 | 0.14 |
| 3 | 0.00 | 0.04 | 0.04 | 0.00 | 0.14 | 0.00 |
| 4 | 0.05 | 0.01 | 0.00 | 0.04 | 0.09 | 0.04 |

We can describe a distance in pretty much any way we want.

## distance matrices

$$
\text { quadratic difference: } \mathrm{C}^{*}\left(\hat{e}\left(\boldsymbol{x}_{\boldsymbol{i}}\right)-\hat{e}\left(\boldsymbol{x}_{\boldsymbol{j}}\right)\right)^{2}
$$

| obs | $e^{\wedge}(\mathbf{x})$ |
| ---: | ---: |
| 1 | 0.54 |
| 2 | 0.43 |
| 3 | 0.57 |
| 4 | 0.53 |
| 5 | 0.57 |
| 6 | 0.54 |
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|  | 5 | 6 | 7 | 8 | 9 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  |
| 3 |  |  |  |  |  |  |
|  |  |  |  |  |  | $\checkmark$ |

We can describe a distance in pretty much any way we want.

## distance matrices

## quadratic difference: $\mathrm{C}^{*}\left(\hat{e}\left(\boldsymbol{x}_{\boldsymbol{i}}\right)-\hat{e}\left(\boldsymbol{x}_{\boldsymbol{j}}\right)\right)^{2}$

| obs | $\mathrm{e}^{\wedge}(\mathbf{x})$ |
| ---: | ---: |
| 1 | 0.54 |
| 2 | 0.43 |
| 3 | 0.57 |
| 4 | 0.53 |
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| 6 | 0.54 |
| 7 | 0.53 |
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|  | 5 | 6 | 7 | 8 | 9 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0.20 | 0.00 | 0.02 | 0.17 | 2.32 | 0.19 |
| 2 | 4.08 | 2.26 | 2.05 | 3.96 | 0.00 | 4.0 |
| 3 | 0.00 | 0.27 | 0.35 | 0.00 | 3.86 | 0.0 |
|  | 0.41 | 0.01 | 0.00 | 0.37 | 1.76 | 0.40 |

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| ---: | ---: |
| 1 | 0.54 |
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|  | 5 |  | 6 | 7 | 8 | 9 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | , | 0.20 | 0.00 | 0.02 | 0.17 | 2.32 | 0.19 |
| 2 |  | 4.08 | 2.26 | 2.05 | 3.96 | 0.00 | 4.06 |
| 3 |  | 0.00 | 0.27 | 0.35 | 0.00 | 3.86 | 0.00 |
| 4 | , | 0.41 | 0.01 | 0.00 | 0.37 | 1.76 | 0.40 |

## distance matrices

- According to the estimated propensity score, 3 has adequate matches with 5,8 and 10

| obs | b_weight | gest_age | dose | death | $\mathbf{e}^{\wedge}(\mathbf{x})$ |
| ---: | ---: | ---: | ---: | ---: | ---: |
| 1 | 2412 | 36 | 1 | 0 | 0.54 |
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| 6 | 2436 | 35 | 0 | 0 | 0.54 |
| 7 | 2461 | 34 | 0 | 0 | 0.53 |
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| ---: | ---: | ---: | ---: | ---: | ---: |
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| 5 | 2569 | 36 | 0 | 0 | 0.57 |
| 8 | 2759 | 32 | 0 | 0 | 0.57 |
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| ---: | ---: | ---: | ---: | ---: | ---: |
| 3 | 2569 | 36 | 1 | 0 | 0.57 |
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- While it's clear that 5 is best, because of the exact match, we can probably rank the match quality as 5,10 , followed by 8 .
- It'd be nice to have a method for taking account of both the p-score and individual level covariates.


## distance matrices

- One solution: use Mahalanobis distance


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- If we write the sample covariance matrix of $\boldsymbol{x}$ as $\hat{\Sigma}$ and $\boldsymbol{x}_{i}$ and $\boldsymbol{x}_{j}$ as the covariate vectors for observations $i$ and $j$, then the Mahalanobis distance between $i$ and $j$ is:

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\left(x_{i}-x_{j}\right)^{T} \widehat{\Sigma}^{-1}\left(x_{i}-x_{j}\right)
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$$

- Intuition:
- Trying to weight each variable equally
- It was one of the first distances used by the matching community
- Mahalanobis distance was created with iid Normals in mind
- Not great at dealing with highly correlated covariates
- Not great at dealing with non-symmetric data


## distance matrices

Mahalanobis distance

| obs | b_weight | gest_age |
| ---: | ---: | ---: |
| 1 | 2412 | 36 |
| 2 | 2205 | 29 |
| 3 | 2569 | 36 |
| 4 | 2443 | 34 |
| 5 | 2569 | 36 |
| 6 | 2436 | 35 |
| 7 | 2461 | 34 |
| 8 | 2759 | 32 |
| 9 | 2324 | 27 |
| 10 | 2667 | 34 |


|  | 5 | 6 | 7 | 8 | 9 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $\int$ |  |  |  |  |  |
| 2 |  |  |  |  |  |  |
| 3 |  |  |  |  |  |  |
|  |  |  |  |  |  | - |

## distance matrices

## Mahalanobis distance

| obs | b_weight | gest_age |
| ---: | ---: | ---: |
| 1 | 2412 | 36 |
| 2 | 2205 | 29 |
| 3 | 2569 | 36 |
| 4 | 2443 | 34 |
| 5 | 2569 | 36 |
| 6 | 2436 | 35 |
| 7 | 2461 | 34 |
| 8 | 2759 | 32 |
| 9 | 2324 | 27 |
| 10 | 2667 | 34 |


| 5$\quad 6$ |
| :--- |
| $\mathbf{1}$ |
| $\mathbf{2}$ |
| 2 |
| 3 |
| 4 | |  | 7 | 8 | 9 | 10 |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.37 | 0.16 | 0.31 | 1.12 | 1.02 | 0.74 |
| 0.88 | 0.68 | 0.62 | 1.16 | 0.45 | 0.96 |
| 0.00 | 0.27 | 0.26 | 0.80 | 0.97 | 0.40 |
| 0.28 | 0.13 | 0.04 | 0.88 | 0.76 | 0.52 |

## distance matrices

- Better to default to the rank-based Mahalanobis distance.


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## distance matrices

- Better to default to the rank-based Mahalanobis distance.
- Big picture:
- Why don't we use just Mahalanobis?
- Why not just the pscore?
- IF we take pscore and Mahalanobis together...


## distance matrices

# Why Propensity Scores <br> Should Not Be Used for Matching* 

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#### Abstract

We show that propensity score matching (PSM), an enormously popular method of preprocessing data for causal inference, often accomplishes the opposite of its intended goal - increasing imbalance, inefficiency, model dependence, and bias. PSM supposedly makes it easier to find matches by projecting a large number of covariates to a scalar propensity score and applying a single model to produce an unbiased estimate. However, in observational analysis the data generation process is rarely known and so users typically try many models before choosing one to present. The weakness of PSM comes from its attempts to approximate a completely randomized experiment, rather than, as with other matching methods, a more efficient fully blocked randomized experiment. PSM is thus uniquely blind to the often large portion of imbalance that can be eliminated by approximating full blocking with other matching methods. Moreover, in data balanced enough to approximate complete randomization, either to begin with or after pruning some observations, PSM approximates random matching which, we show, increases imbalance even relative to the original data. Although these results suggest that researchers replace PSM with one of the other available methods when performing matching, propensity scores have many other productive uses.


## distance matrices

## Building a Stronger Instrument in an Observational Study of Perinatal Care for Premature Infants

Mike Balocchi, Dylan S. Small, Scott LORCH, and Paul R. Rosenbaum


1. INTRODUCTION: MOTIVATION EXAMPLE, AND DATA
1.1 Regionalization of Intensive Care for Premature Infants: Does It Save Lives?

Hospitals vary in their ability to care for premature infants. The American Academy of Pediatrics recognizes six levels neonatal intensive care units (NICUs) of increasing technical expertise and capability: $1,2,3 \mathrm{~A}, 3 \mathrm{~B}, 3 \mathrm{C}, 3 \mathrm{D}$, and regional centers, 4. The term "regionalization of care" refers to a policy that with greater capabilities. In other words, within a region, moth ens are to be sorted into bospitals of varied capability based on the risks faced by the newborn, rather than on haphazard cirumstances, such as afthiation or proximity. Regionalized perinatal systems were developed in the 1970s, when NICUs began 0 save infants with birth weight $<1500 \mathrm{~g}$. In the 1990 s , how ever, NICU services began to diffuse from regional centers to community hospitals. Regionalization might reduce infant mor ble hospitals; however, regionalization might not reduce infant mortality because the sorting by risk might be too inaccurate to affect bealth, or the capabilities of high-level NICUs might fail oo deliver better outcomes.
In the current paper, we focus on whether delivering high nisk infants at more capable NICUs reduces mortality. This is one key component in the evaluation of regionalized perinatal systems. More precisely, if a high-risk mother delivers at less capable hospita, is her baby al greater risk of deanh?
a randomized experiment could settle that question, with highrisk mothers assigned at random to hospitals of varied capabilities. In the world that we actually do inhabi,
decisions are happily constrained by considerations of sound judgment, ethics, and patient preferences, such an experiment is not possible. We need to make some reasonable sense of the data that we can obtain. There is a basic difficulty, however, that arises in many contexts in which the most intense and capable care is given to the sickest patients. If regionalization succeeded in sorting mothers by risk, then the highest-risk mothrates at the more-capable bospitals might be higher, not lower. than those the less-capable hospitals because their patient populations were sicker, even if the more-capable hospitals were saving lives. A nalve comparison of mortality rate by level of NICU would do little or nothing to clarify whether regional ization is or is not effective, because it would not estimate the effect on mortality of delivery at a more-capable hospital Here we take an old tactic and improve it. The old tactic exa hospital with a high-level NICU if such a bospital is close to home. A pregnancy may conclude with a certain urgency, and awareness of this possibility may lead the mother to want to avoid a long trip. If travel time to a bospital with a high-level NICU affected risk only if it altered whether the baby received care at that hospital, then the so-called "exclusion restriction" would be plausible. (See Angrist, Imbens, and Rubin 1996 for a discussion of the exclusion restriction.) If it were also true that would be an instrument for care at a hospital with high-leyel

Using the Prognostic Score to Reduce Heterogeneity in Observational Studies

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## Abstract

In large sample observational studies, the control population often greatly outnumbers the treatment population. Typical practice is to match several control observations to a single treated observation, with the goal of reducing sampling variability of the resulting treatment effect estimate. However, increasing the control to treated ratio yields diminishing returns in terms of variance reduction and in practice leads to poorer quality matches. In line with inferencem sargument on the importance reducing heterogeneity to strengthen ca to fit a prognostic model, then match on the resulting prognostic score to create matched sets with lower heterogeneity. We propose methodological alternatives to fitting the prognostic model that help avoid concerns of overfitting and extrapolation, then demonstrate in a simulation setting how this alternative use of the control observations can lead to gains in terms of both treatment effect estimation and design sensitivity
, propensity score, prognostic score

1. Introduction

Unlike in a randomized experiment, any claim of a causal effect based on observational data must address the possibility of bias due to non-random treatment assignment. Matching methods attempt to adjust for this bias by recreating a randomized experiment, grouping reatment and control subjects in a way that balances the observed covariate distributions ?). However, matching does not guarantee balance in the unobserved covariates - practiioners typically carry out their analyses making the unverifiable assumption that all the elevant covariates have been observed. Performing a sensitivity analysis provides a way to

## penalty functions

- Add on a caliper using the pscore


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## penalty functions

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- Penalty functions for pscore


## penalty functions

## Mahalanobis distance

| obs | b_weight | gest_age |
| ---: | ---: | ---: |
| 1 | 2412 | 36 |
| 2 | 2205 | 29 |
| 3 | 2569 | 36 |
| 4 | 2443 | 34 |
| 5 | 2569 | 36 |
| 6 | 2436 | 35 |
| 7 | 2461 | 34 |
| 8 | 2759 | 32 |
| 9 | 2324 | 27 |
| 10 | 2667 | 34 |


|  | 5 | 6 | 7 | 8 | 9 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0.37 | 0.16 | 0.31 | 1.12 | 1.02 | 0.74 |
| 2 | 0.88 | 0.68 | 0.62 | 1.16 | 0.45 | 0.9 |
| 3 | 0.00 | 0.27 | 0.26 | 0.80 | 0.97 | 0.4 |
|  | 0.28 | 0.13 | 0.04 | 0.88 | 0.76 | 0.52 |

## penalty functions

$\underline{\text { absolute difference: }\left|\hat{e}\left(\boldsymbol{x}_{i}\right)-\hat{e}\left(\boldsymbol{x}_{j}\right)\right|}$

| obs | $e^{\wedge}(\mathbf{x})$ |
| ---: | ---: |
| 1 | 0.54 |
| 2 | 0.43 |
| 3 | 0.57 |
| 4 | 0.53 |
| 5 | 0.57 |
| 6 | 0.54 |
| 7 | 0.53 |
| 8 | 0.57 |
| 9 | 0.43 |
| 10 | 0.57 |


| 5 <br> 5 $6^{\prime}$ |
| :---: |
| 1 |
| 1 |
| 2 |
| 3 |
| 4 | | 0.03 | 0.00 | 0.01 | 0.03 | 0.11 | 0.03 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.14 | 0.11 | 0.10 | 0.14 | 0.00 | 0.14 |
| 0.00 | 0.04 | 0.04 | 0.00 | 0.14 | 0.00 |
| 0.05 | 0.01 | 0.00 | 0.04 | 0.09 | 0.04 |

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| :--- |
| $\mathbf{1}$ |
| $\mathbf{1}$ |
| 2 |
| 3 |
| 4 | | 0.37 | 0.16 | 0.31 | 1.12 | 1.02 | 0.74 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.88 | 0.68 | 0.62 | 1.16 | 0.45 | 0.96 |
| 0.00 | 0.27 | 0.26 | 0.80 | 0.97 | 0.40 |
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| :--- | :--- | :--- | :--- | :--- | :--- |
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## penalty functions



This caliper sets the distance matrix to infinity if $\left|\hat{e}\left(\boldsymbol{x}_{i}\right)-\hat{e}\left(\boldsymbol{x}_{j}\right)\right| \geq 0.10$

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where $M_{M}$ is from the Mahalanobis $M_{p}$ is a penalty based on the pscores and $M_{d}$ is the distance matrix that will be used to match.

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You can add in a penalty, such as $\mathrm{C}^{*}\left(\hat{e}\left(\boldsymbol{x}_{i}\right)-\hat{e}\left(\boldsymbol{x}_{j}\right)\right)^{2}$, when $\left|\hat{e}\left(\boldsymbol{x}_{i}\right)-\hat{e}\left(\boldsymbol{x}_{j}\right)\right|$ are outside of some acceptable range.

## penalty functions: covariates



You can add in a penalty, such as $\mathrm{C}^{*}\left(x_{i, g e s t \_a g e}-x_{j, \text { gest_age }}\right)^{2}$, when $\left|x_{i, g e s t \_a g e}-x_{j, g e s t \_a g e}\right|$ are outside of some acceptable range.

## penalty functions: covariates



You can also do a one-side penalty function to nudge in one direction. For example when $\left(x_{i, g e s t \_a g e}-x_{j, \text { gest_age }}\right) \leq-1$.

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- Think about both the individual and the group level.
- Individual level for matched-pairs randomization
- Group level for the comparability of the two groups (e.g., "Table 1")


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- In prospective studies
design is an obvious consideration
- and one that MUST be passed through in order to obtain data
- In retrospective studies,
- design is a less obvious consideration
- but one that MUST be passed through... unfortunately without much attention paid
fin.


## fin.

## CHECK OUT THE WEBSITE.

